

# Zuur Ch 02 part 2

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# Ch 02 outline

- Learn as you go philosophy
- Data exploration
- Linear regression modelling
- Linear regression assumptions
- Examples

## 2.3 Violate those assumptions

Vanilla linear model full assumptions

- Gaussian residuals
- Homogeneous variance
- “fixed”  $X$  (discuss briefly)
- Independence
- Correct model specification...

## 2.3.6 wedge clams



## 2.3.6 wedge clams

```
Clams <- read.table("Clams.txt", header = T)
str(Clams)
```

```
## 'data.frame':   398 obs. of  5 variables:
## $ MONTH      : num  11 11 11 11 11 11 11 11 11 11 ...
## $ LENGTH     : num  28.4 16.6 13.7 17.4 11.8 ...
## $ AFD        : num  0.248 0.052 0.028 0.07 0.022 0.187 0.361 0.05 0.087 0.128 ...
## $ LNLENGTH: num  3.35 2.81 2.62 2.86 2.47 ...
## $ LNAFD      : num  -1.39 -2.96 -3.57 -2.65 -3.83 ...
```

Month - month of measurement

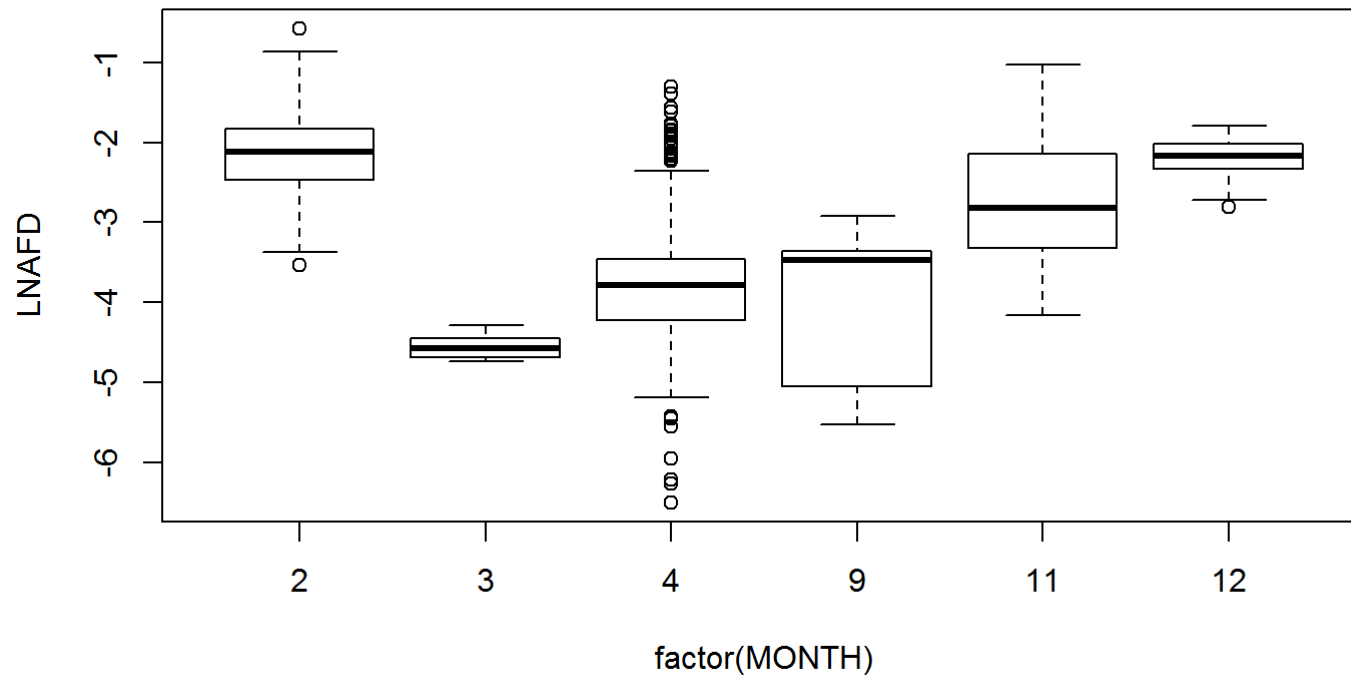
Length - length (mm?)

AFD - weight

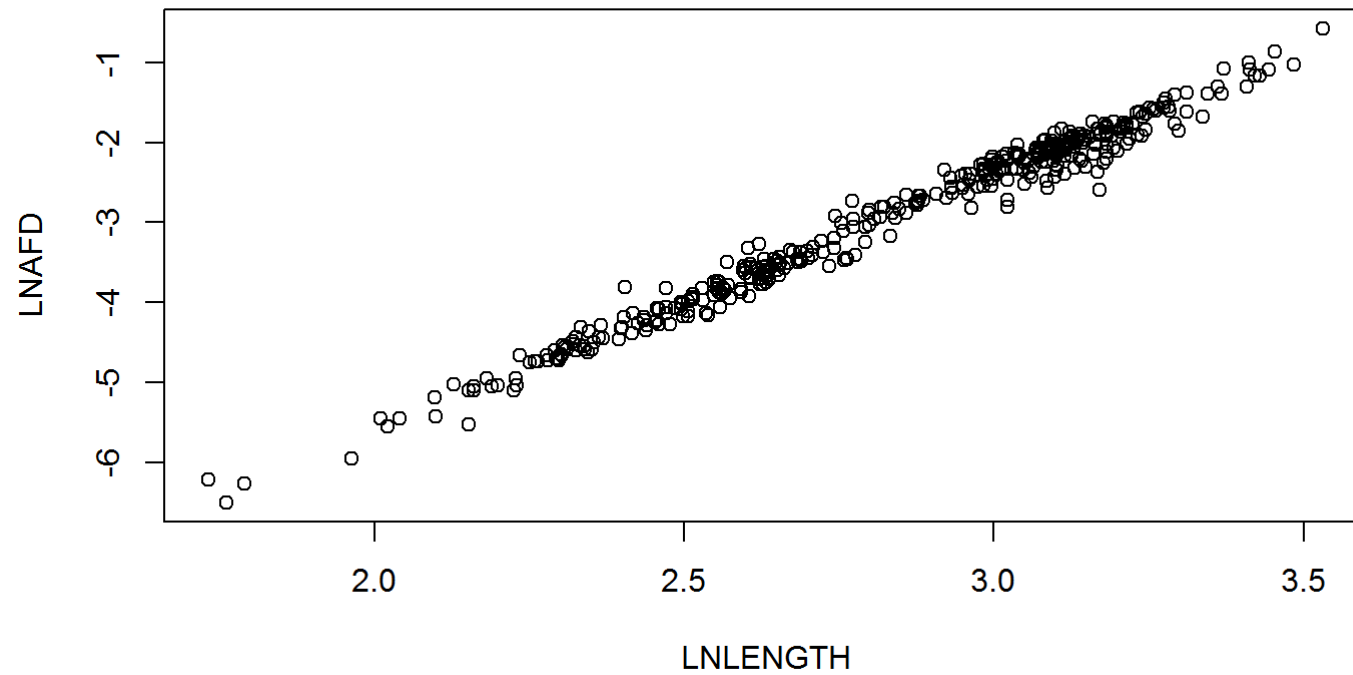
LNLENGTH - log(Length)

LMAFD - log(AFD)

## 2.3.6 wedge clams



## 2.3.6 wedge clams



## 2.3.6 wedge clams

“possible models”:

$LNAFN \sim LNLENGTH + MONTH$

$LNAFN \sim LNLENGTH * MONTH$

...#isThisOK? **Discuss**

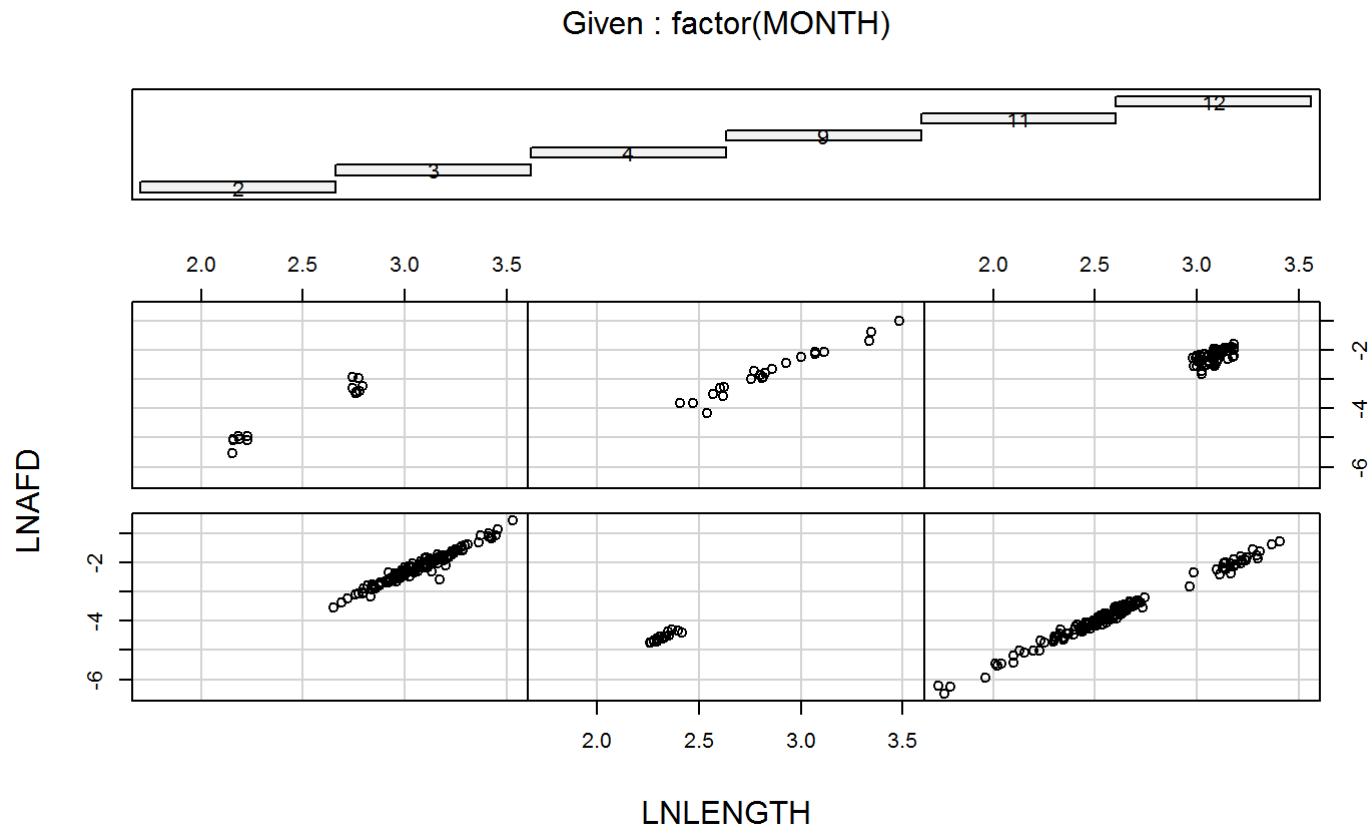


# Reading for model selection

Aho, K., Derryberry, D., Peterson, T., 2014. Model selection for ecologists: the worldviews of AIC and BIC. *Ecology* 95, 631-636. <https://doi.org/10.1890/13-1452.1>

# Testing alternative models

Month:  
9 11 12  
2 3 4



# Testing alternative models

```
## Single term deletions
##
## Model:
## LNAFD ~ LNLENGTH * factor(MONTH)
##
##           Df Sum of Sq    RSS      AIC F value  Pr(>F)
## <none>                6.4490 -1616.8
## LNLENGTH:factor(MONTH)  5   0.20328 6.6523 -1614.4  2.4334 0.03444 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Which model is best?

## 2.3.6 wedge clam model validation

Do this for your models:

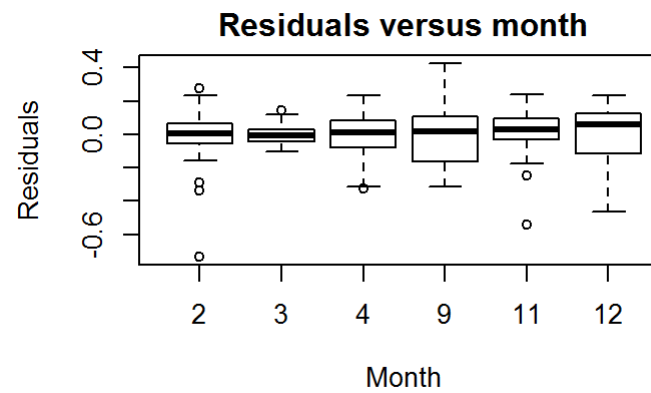
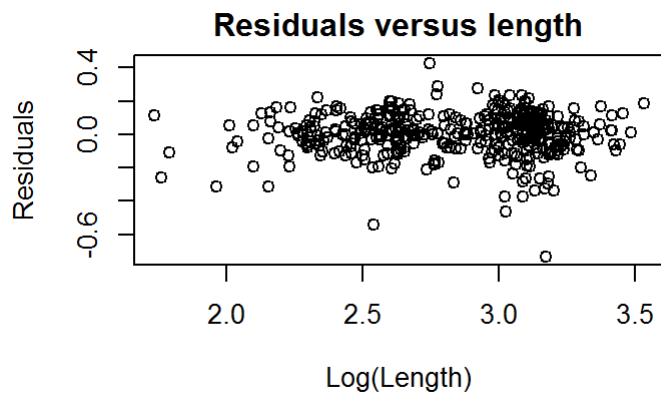
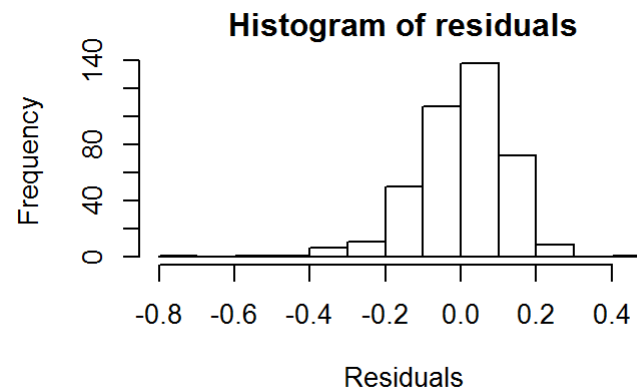
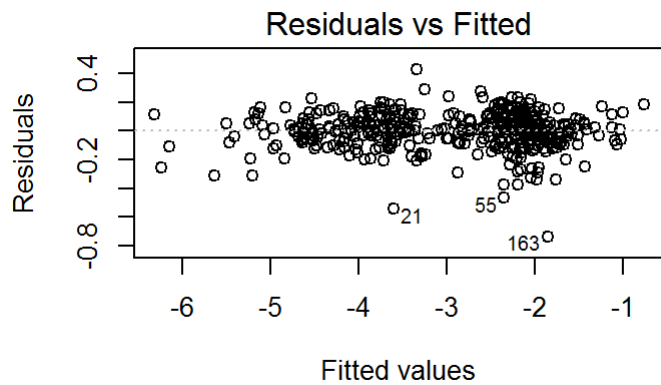
1. Residual versus fitted values  
(looking for homogeneity of variance)  
((heteroskedasticity)) \*spelling
2. Residual distribution  
(qq plot or histogram, etc.)
3. Residuals versus explanatory variables  
(is residual variation independent of  
variation in explanatory vars?)

# Heteroskedasticity

Paloyo, A.R., 2011. When Did We Begin to Spell "Heteros\*Edasticity" Correctly? (SSRN Scholarly Paper No. ID 1973444). Social Science Research Network, Rochester, NY. <https://doi.org/10.2139/ssrn.1973444>

# Test of assumptions

Discuss



# Test of assumptions

Formal tests of homogeneity of residuals

E.g. F-test (Faraway 2005) does variance differ by clam length?

```
##  
## F test to compare two variances  
##  
## data: E1 and E2  
## F = 0.67571, num df = 75, denom df = 321, p-value = 0.04211  
## alternative hypothesis: true ratio of variances is not equal to 1  
## 95 percent confidence interval:  
## 0.4821187 0.9858005  
## sample estimates:  
## ratio of variances  
## 0.6757124
```

# Test of assumptions

Formal tests of homogeneity of residuals

Bartlett test does variance differ by month?

```
##  
## Bartlett test of homogeneity of variances  
##  
## data: E and factor(Clams$MONTH)  
## Bartlett's K-squared = 37.017, df = 5, p-value = 5.942e-07
```



# Results

We have some reasons to be unsatisfied with the “plain old” linear model

```
## Analysis of Variance Table
##
## Response: LNAFD
##
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
## LNLENGTH	1	480.86	480.86	28781.3185	< 2e-16	***
## factor(MONTH)	5	2.04	0.41	24.4282	< 2e-16	***
## LNLENGTH:factor(MONTH)	5	0.20	0.04	2.4334	0.03444	*
## Residuals	386	6.45	0.02			

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## 2.3.7 Moby's Teeth

Sperm whales beach and die. Famous Moby on Firth of Forth. Dataset looking at Nitrogen isotope accumulation in teeth by age.



## 2.3.7 Moby's Teeth

## 2.3.7 Moby's Teeth

```
Teeth <- read.table("TeethNitrogen.txt", header = T)
str(Teeth)
```

```
## 'data.frame': 307 obs. of 3 variables:
## $ X15N : num 11.7 11.7 11.5 11.7 11.7 ...
## $ Age : int 1 2 3 4 5 6 7 8 9 10 ...
## $ Tooth: Factor w/ 11 levels "I1/98","M143/96D",...: 7 7 7 7 7 7 7 7 7 7 ...
```

## 2.3.7 Moby's Teeth

Model (just Moby's)  $N \sim \text{age} \dots$

```
M2 <- lm(X15N ~ Age, data = Teeth,  
        subset = (Teeth$Tooth == "Moby"))  
anova(M2)
```

```
## Analysis of Variance Table
```

```
##
```

```
## Response: X15N
```

```
##           Df Sum Sq Mean Sq F value    Pr(>F)
```

```
## Age         1  79.902   79.902  338.43 < 2.2e-16 ***
```

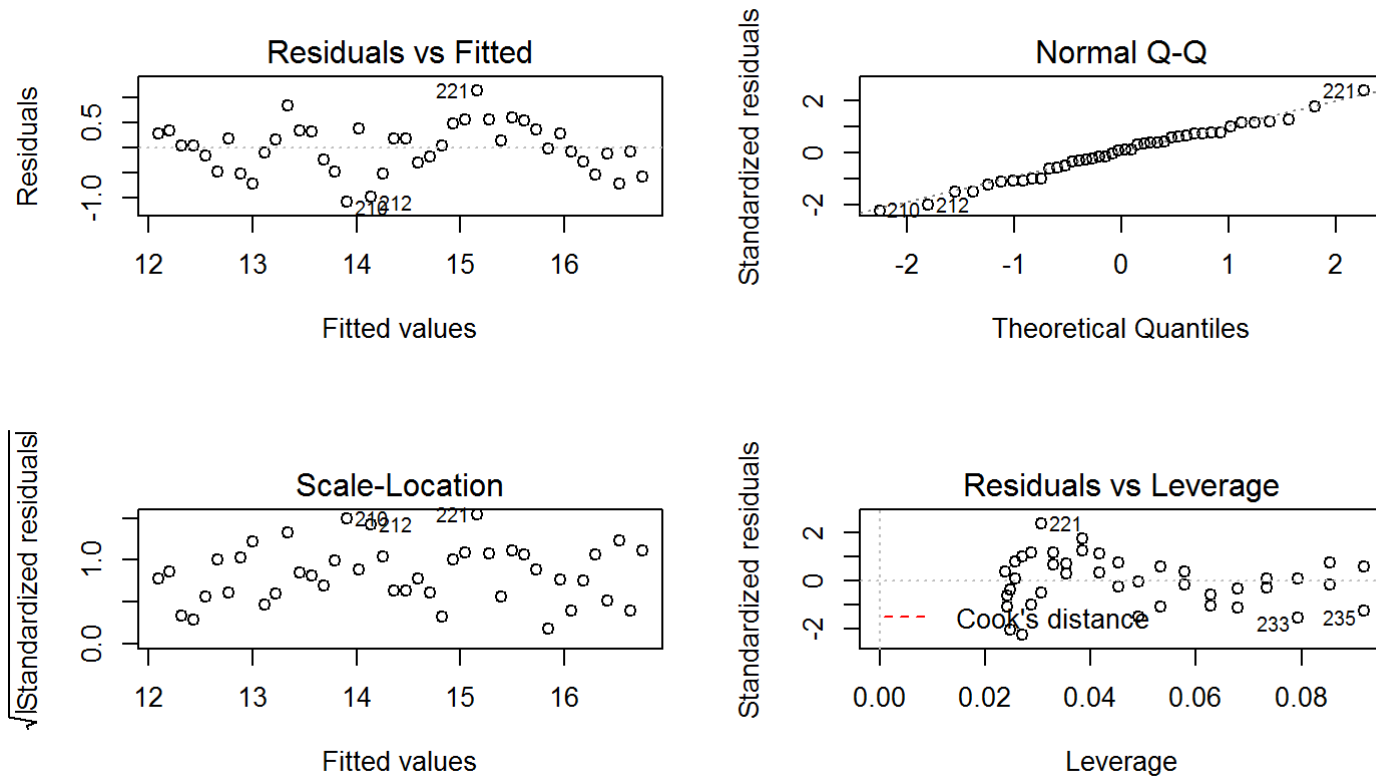
```
## Residuals 40   9.444    0.236
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

## 2.3.7 Moby's Teeth

Regular diagnostic plots available in R by using `plot()` on a linear model



object

## 2.3.7 Moby's Teeth

NB - Striking pattern of residuals by case! - Normal qq plot (discuss) looks okay - Cook's distance plot (looking for values outside red line - none here) ((measure of "high leverage" outliers to consider excluding))

Fox, J., 2015. Applied Regression Analysis and Generalized Linear Models, 3rd ed. SAGE Publications, Los Angeles.

## 2.3.7 Moby's Teeth

Violation of homogeneity and non-independence...

```
justMoby <- which(Teeth$Tooth == "Moby")  
plot(X15N ~ Age, data = Teeth[justMoby, ])  
abline(lm(X15N ~ Age, data = Teeth[justMoby, ]))
```



## 2.3.7 Moby's Teeth

Accept or reject this model?

$$y_i = 11.75 + 0.11 \times \text{age}_i$$

```
##  
## Call:  
## lm(formula = X15N ~ Age, data = Teeth[justMoby, ])  
##  
## Residuals:  
##      Min       1Q   Median       3Q      Max   
## -1.07102 -0.28706  0.04346  0.33820  1.13724   
##  
## Coefficients:  
##              Estimate Std. Error t value Pr(>|t|)      
## (Intercept) 11.748940   0.163559   71.83  <2e-16 ***   
## Age          0.113794   0.006186   18.40  <2e-16 ***   
## ---  
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  
##  
## Residual standard error: 0.4859 on 40 degrees of freedom
```

# Homework :)

For the Nereis dataset: -run the model concentration ~ biomass \* nutrients - evaluate assumptions -come prepared to discuss it with evidence for next time